Mathematical Problems in Networked Embedded Systems

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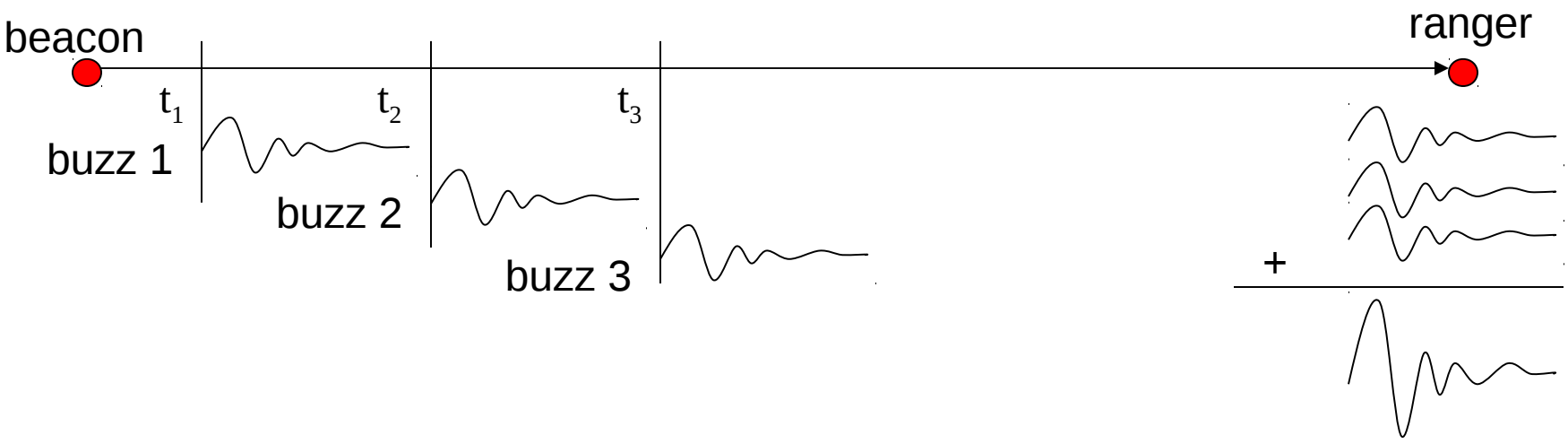
Outline

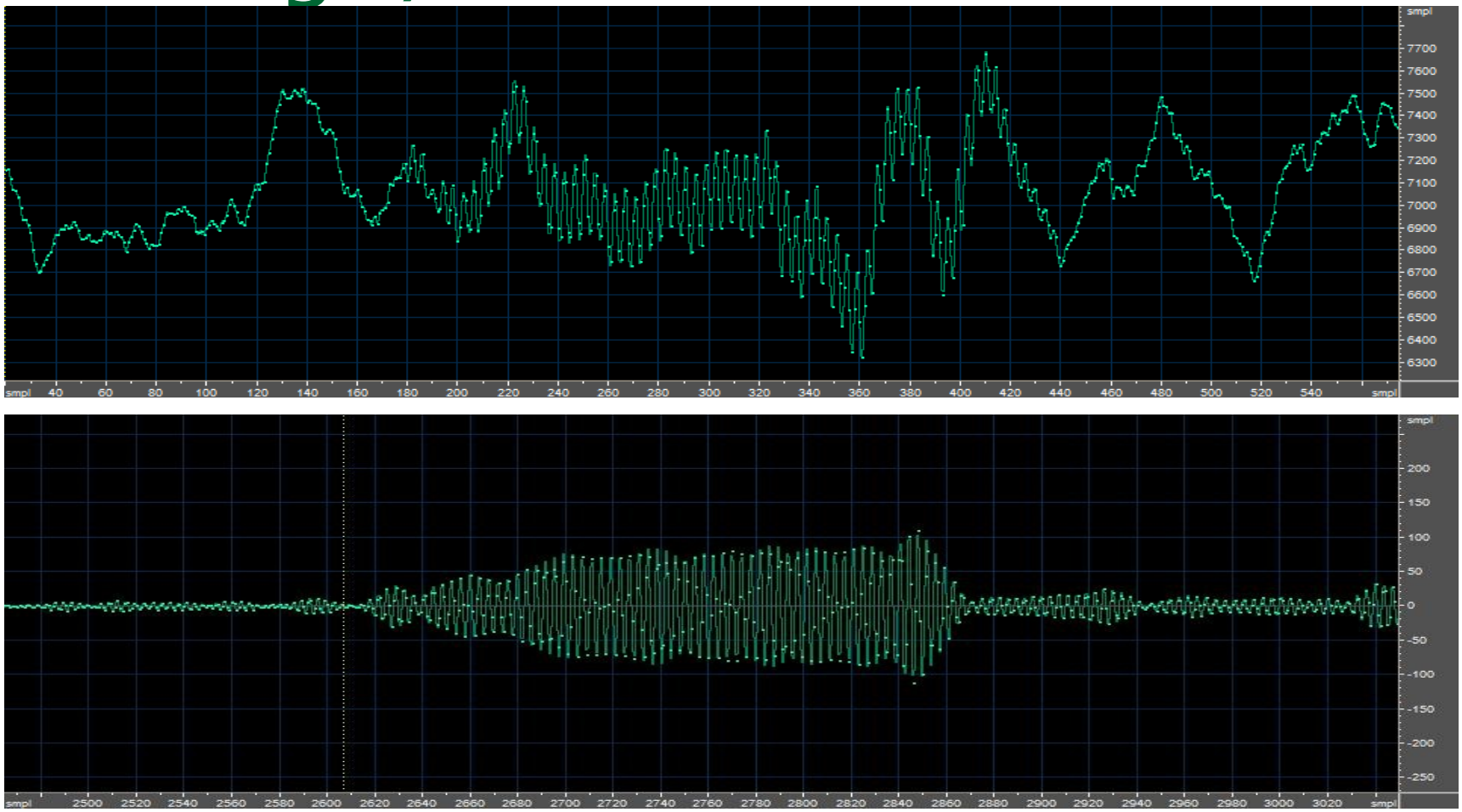
- Acoustic ranging
- TDMA in globally asynchronous locally synchronous networks
- Geographic routing

Acoustic ranging

- Simple idea
  - “beacon” emits radio signal followed by acoustic signal
  - “ranger” measures time difference of arrival, calculates distance
- Modification to improve the signal to noise ratio
  - “beacon” emits several audio signals
  - “ranger” records them, aligns the recorded streams and takes the average



Example (6 meters, 16x average)

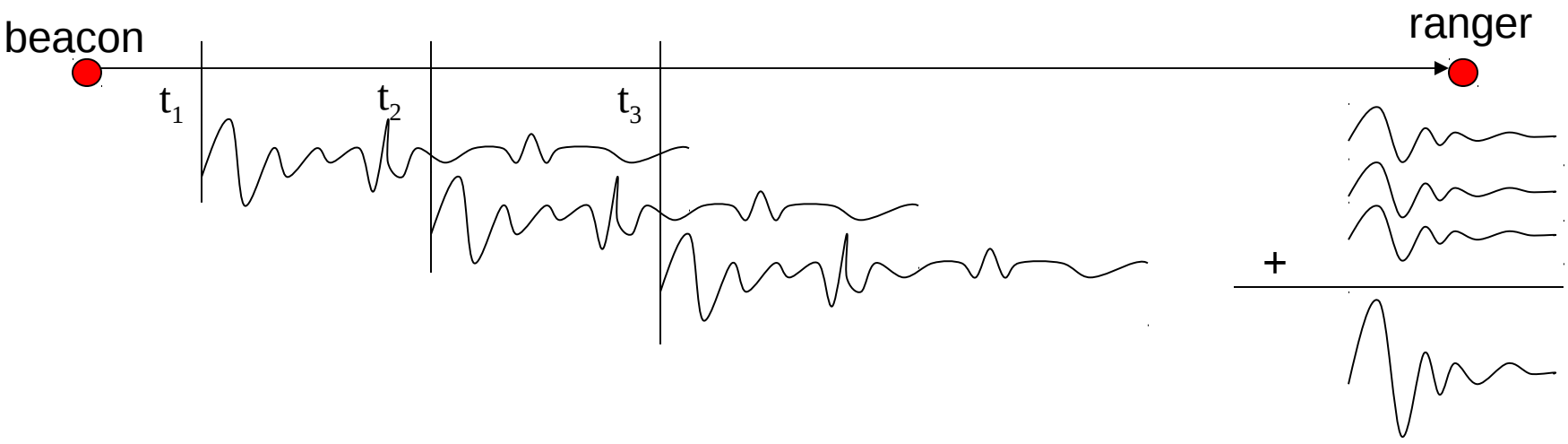


Challenges and requirements

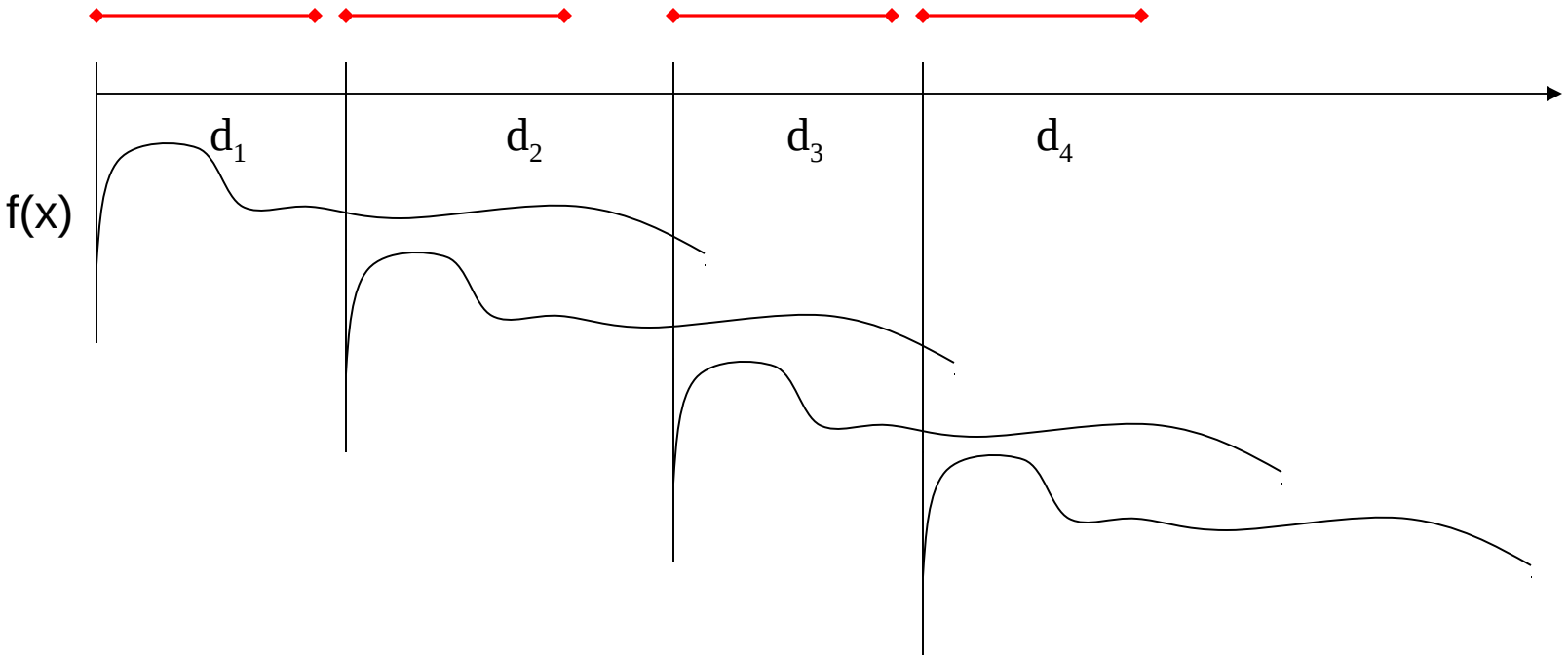
- “beacon” and “ranger” need a common synchronization time instant
  - use time synchronization
  - or just the radio signal (message)
- “beacon” must “buzz” at known times
  - timing at the microcontroller
  - physical delay of the buzzer
- “ranger” must sample the microphone at a known frequency
- “ranger” must start sampling at known time(s), and must be able to align the sampled chirps
  - when does the physical sampling start?
  - what is the time delay between samples?

Echoes and buzz overlaps

- We want to record 60 ms intervals16 times
  - 20 meters of sound
  - Total time is 1 second
- Echoes can be present for up to 300 ms
  - The echoes of the buzzes overlap with the direct signal
  - We can vary the buzz times



Choosing the buzz times

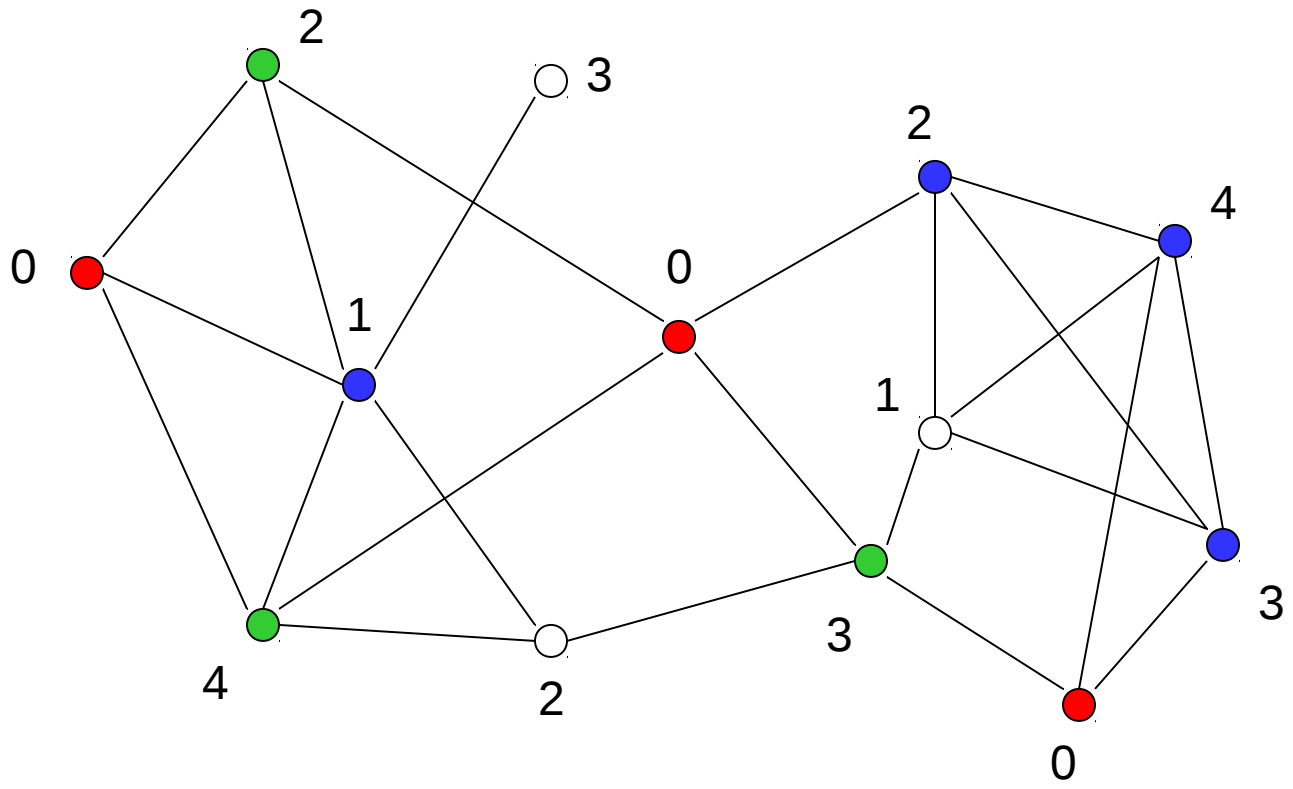


$$g(x) = n \cdot f(x) + \sum_{i=1}^{n-1} f(x + d_i) + \sum_{i=1}^{n-2} f(x + d_i + d_{i+1}) + \cdots$$

Globally Asynchronous Locally Synchronous Networks

- Cooperative wireless MAC
- We have time synchronization in a small part of the network (up to a few hops)
- We have a transmission schedule: time division multiple access (TDMA)
- Assume symmetric and reliable links and a static network
- Represent the network with a graph. The transmission schedule is a coloring.

Collisions



- transmit
- receive
- collision

period: 5

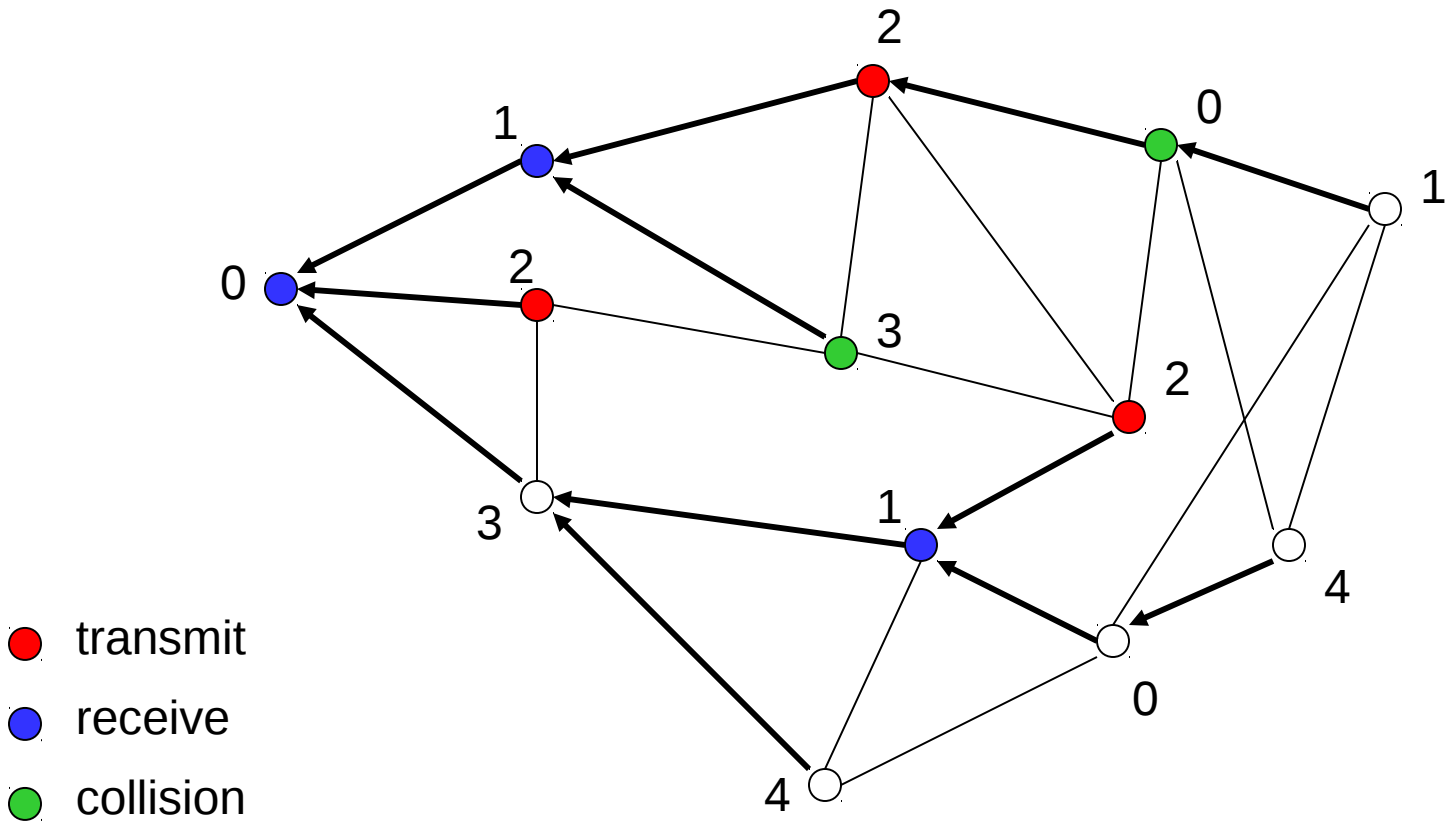
Collision model

- A node receives a message if and only if it is not transmitting and exactly one of its neighbors is transmitting
- The transmission schedule is a good coloring of the 2-transition of the graph
- **Fact:** We can always schedule with period d^2+1 where d is the maximum degree

Converge-cast (beamformation)

- We want to send all messages to a central node in the synchronized subgraph
- Not all collisions are bad
- **Theorem:** We can always schedule the converge-cast with period $d+1$
- **Fact:** There exist arbitrary large graphs with degree d where the optimal converge-cast schedule has period $d/2$
- Good streaming properties

Converge-cast scheduling



New wireless model

- In reality networks are not graphs
  - unidirectional connections
  - connections are not “black” and “white”
  - How to measure the reliability of the connection
- New collision model
  - Message is received if there exits one transmitter whose transmission energy is larger than the sum of the other transmitters

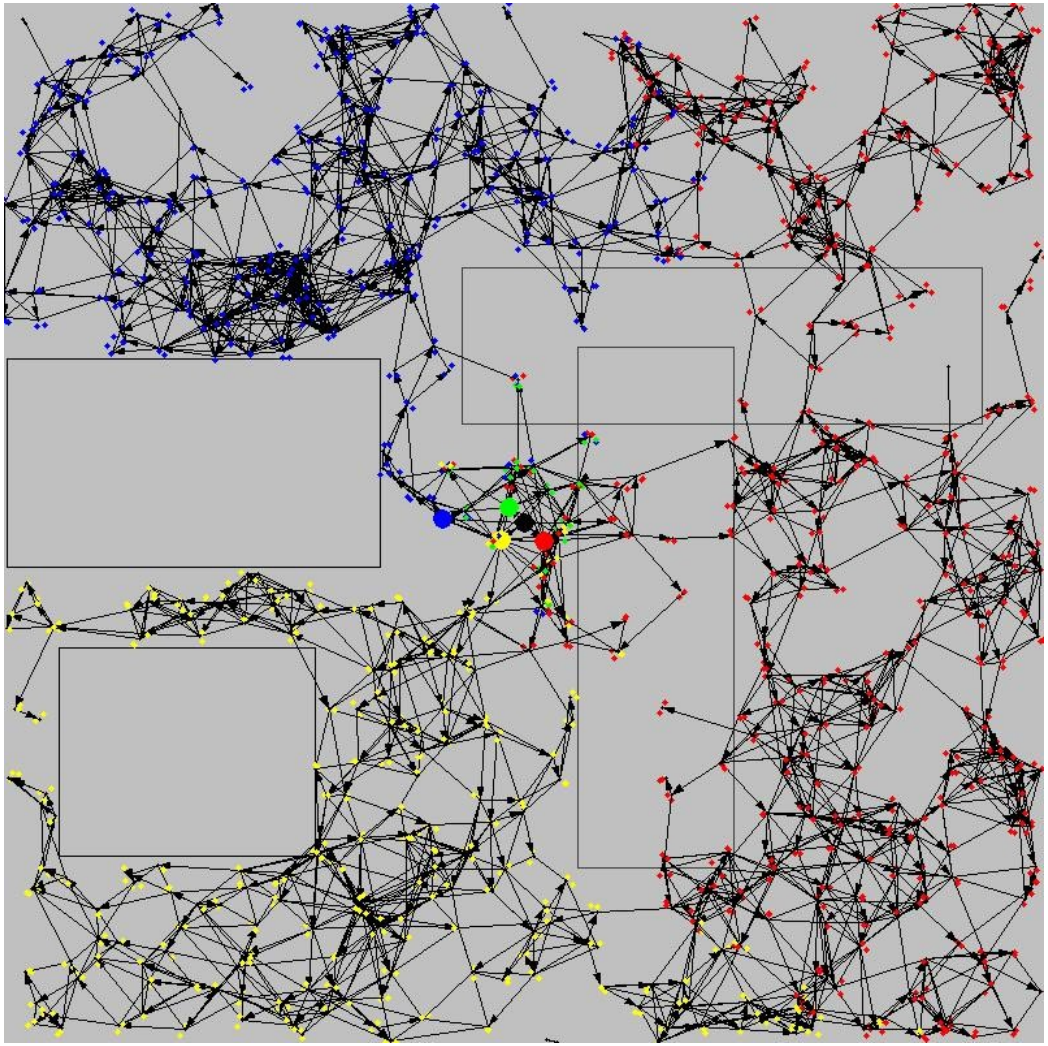
Geographic routing

- Problem: large wireless network, every node knows its own location. We want to send a message to a known location
- Routing step: we have the packet, we know its destination. Which of our neighbors should be the next hop?
- Idea: Each node groups the target locations by the next hop in the shortest path.

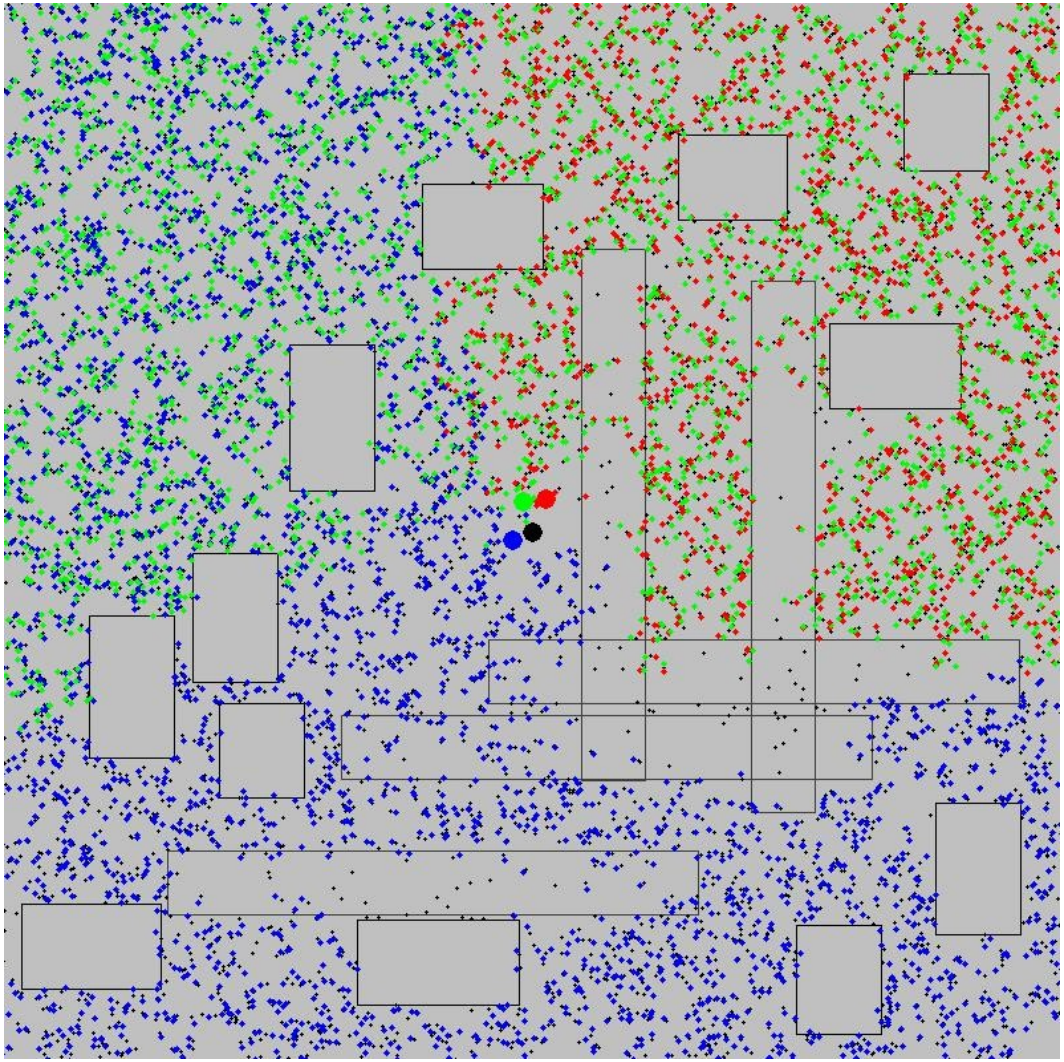
Simulation

- Radio strength (CC1000) from -10dB to 20dB
- Radio sensitivity -110 dB
- Radio signal strength loss exponent
  - Ideal exponent: 2
  - Highly cluttered (buildings, walls): 3.5
  - Random noise: 30 dB
  - Unidirectional
- Buildings: from 40x40 to 40x80 meters
- Road width: 30 meters (20% density)

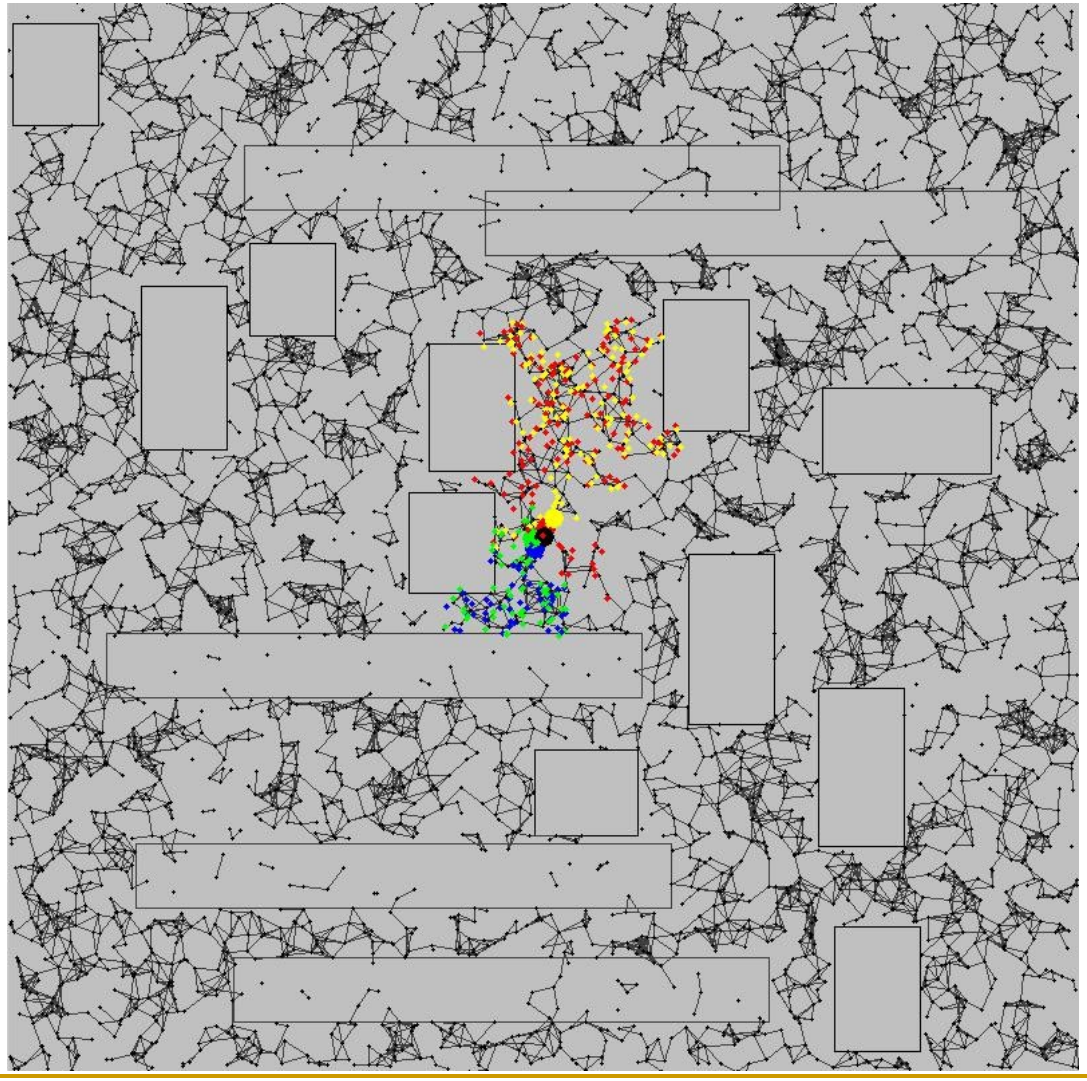
200x200 meters, 0dB, 680 nodes



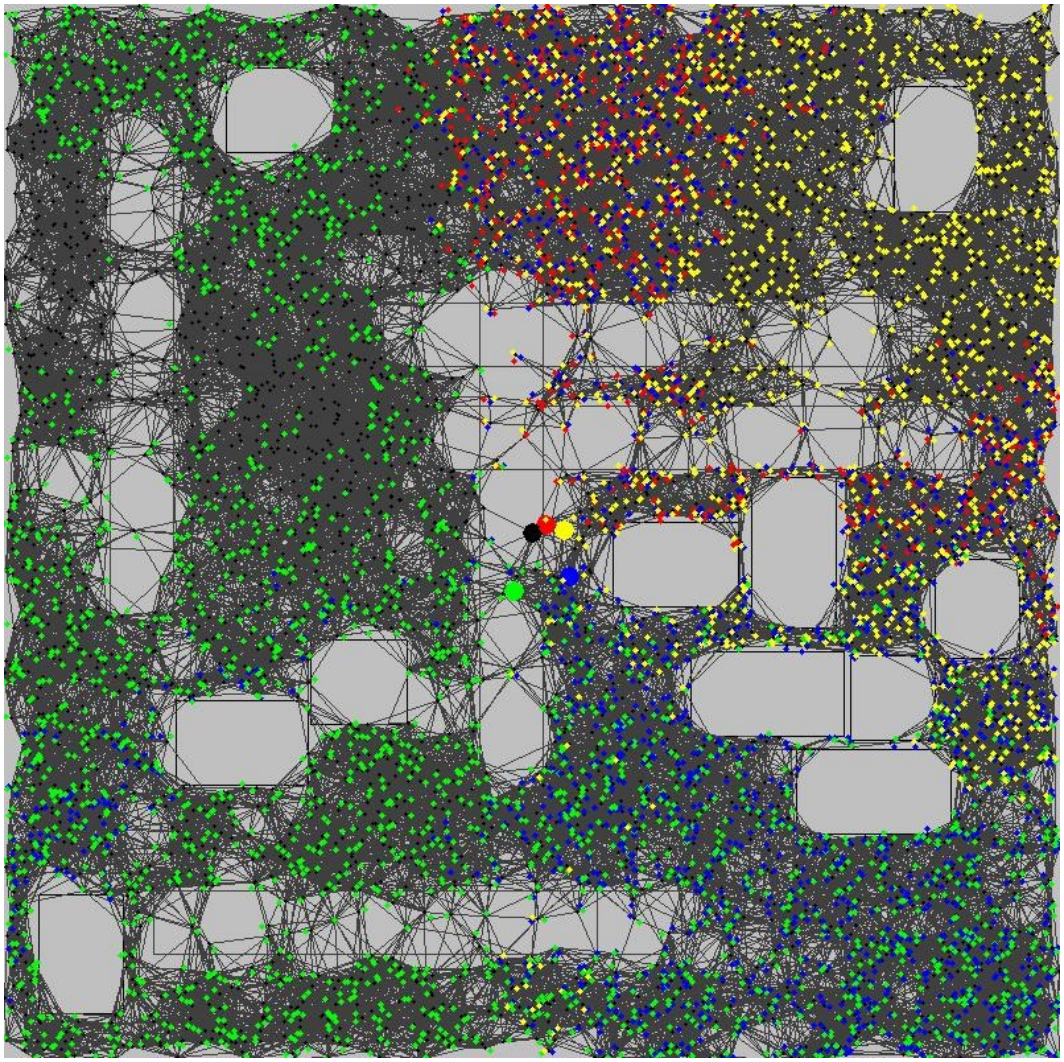
500x500 meter, 4250 nodes, 0dB



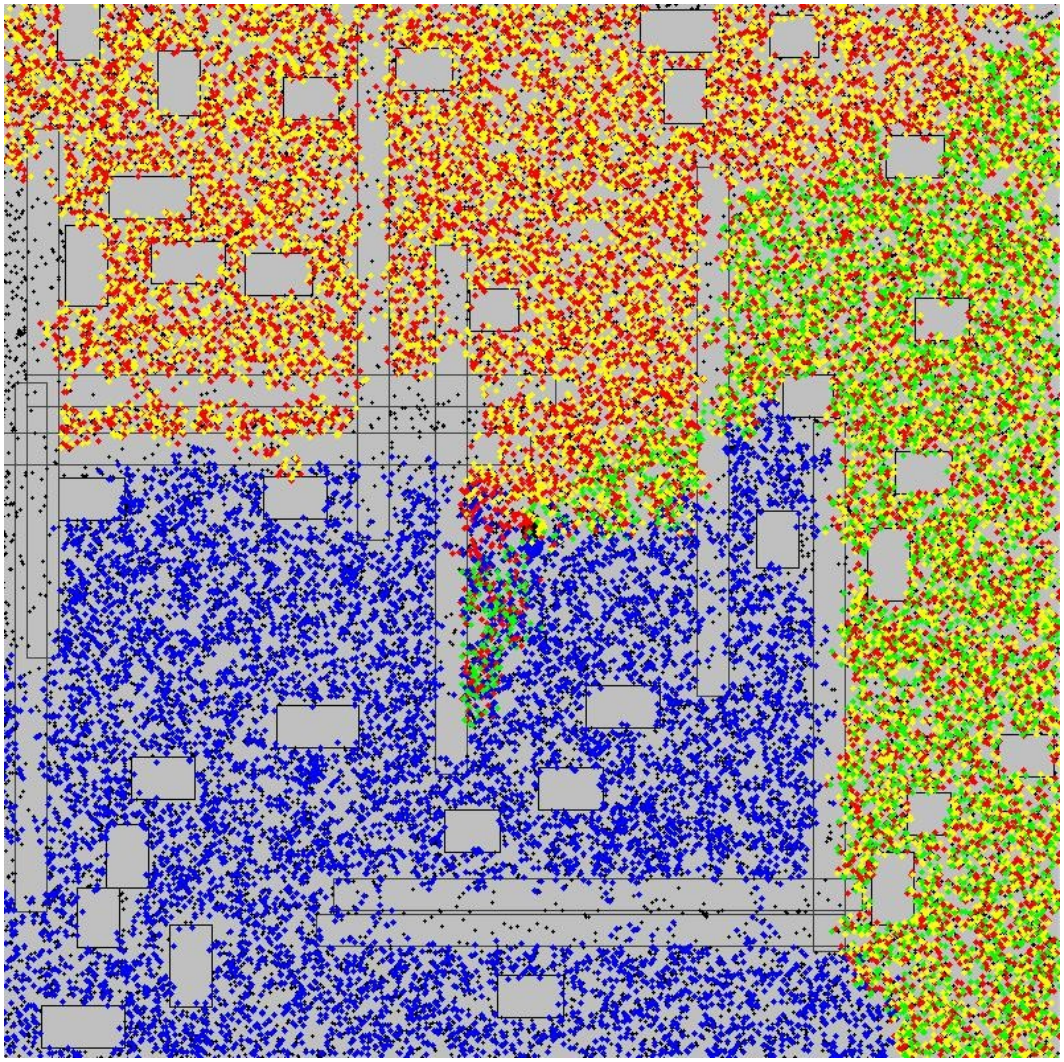
500x500 meter, 4250 nodes, -10dB



500x500 meter, 4250 nodes, 20dB



1000x1000 meters, 17000 nodes, 0dB



Geographic Routing Observations

- Use different radio strengths
  - Minimum strength: the network is not connected
  - Maximum strength: highly connected
- The next hop should be
  - On the macro scale: in the direction of the target
  - On the micro scale: have to go around obstacles
  - Go towards the most connected parts (highway)
  - If cannot find next hop, use high radio strength
- What is a typical sensor network with large number of nodes?